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COMMENT

Comments on 'On a proposed new test of Heisenberg's principle'

D Home and S Sengupta
Solid State Physics Research Centre, Department of Physics, Presidency College, Calcutta–700 073, India

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Abstract. We point out a logical fallacy in Robinson's analysis of a thought experiment purporting to show violation of Heisenberg's uncertainty principle. The real problem concerning the interpretation of Heisenberg's principle is precisely stated.

In a recent paper, Robinson (1980) has analysed a thought experiment which, he has claimed, can lead to a violation of Heisenberg's uncertainty principle $\Delta x \Delta p_x \geqslant \hbar/2$. However, there is a logical fallacy in Robinson's argument, which we shall point out in this note. We first enumerate briefly the essential aspects in Robinson's scheme of the thought experiment.

Detector D_1 at x_1, coupled with a velocity selector, selects an ensemble of particles with velocity along the x axis in the range $v_s - \delta v_s$ to $v_s + \delta v_s$. Thus for the ensemble prepared in this way the momentum uncertainty Δp_x is given by

$$\Delta p_x = m \Delta v_s < m \delta v_s \tag{1}$$

where Δv_s is the RMS deviation in velocity. To demonstrate violation of the uncertainty principle Robinson considers experimental realisation of the possibility $v_t = v_s$ where v_t is the flight velocity of the particles after they have emerged from D_1. Then for the position of any such particle at instant $t > t_1$ (t_1 being the instant the particle has emerged from D_1) we can write

$$x(t) = x_1 + v_s(t - t_1) \tag{2}$$

whence Robinson obtains

$$\Delta x < \delta v_s (t_2 - t_1) \tag{3}$$

assuming

$$\Delta x_1,\ v_s \Delta t_1 \ll \delta v_s (t - t_1) \tag{4}$$

while t_2 is the instant of detection of such a particle at x_2.

Then Robinson takes the crucial step by asserting that 'since in principle $\delta v_s (t_2 - t_1)$ can be made arbitrarily small' one can design an experiment such that

$$\Delta x \Delta p_x < m (\delta v_s)^2 (t_2 - t_1) < \hbar/2 \tag{5}$$

which evidently violates the uncertainty principle.

To pinpoint the snag in the above assertion let us apply the uncertainty relation at the stage the ensemble is prepared. Then corresponding to equation (1) we have

$$\Delta x_1 > \hbar/(2m\delta v_s). \tag{6}$$

The relation (6) together with the assumption (4) implies

$$m(\delta v_s)^2(t_2 - t_1) \gg \hbar/2. \tag{7}$$

Thus it is clear that while writing (5) Robinson has in fact assumed an experimental design which contradicts the restriction (7) imposed by the uncertainty relation; hence in demonstrating violation of the uncertainty relation Robinson has committed the logical fallacy *petitio principii.*

Now, as a sequel to Robinson's remarks highlighting confusion about the 'true' meaning of the uncertainty principle, we wish to observe the following relevant aspects. Rigorous derivation of the uncertainty relation (see for example Merzbacher 1970), based on the mathematical formalism of quantum mechanics, unambiguously implies the following operationally meaningful interpretation of the uncertainty principle.

(i) For any ensemble of identically prepared systems, the product of the RMS deviations of two canonically conjugate variables has a lower bound given by $\hbar/2$.

Indeterminacy in the observed value of a dynamical variable, as referred to in the uncertainty principle, essentially implies the statistical spread in the measured values over the various identical members of the ensemble and this is inherent in any method of preparation of an ensemble. The essence of the uncertainty principle lies in ruling out the possibility of preparing an ensemble of identical particles in the same state where the product of the RMS deviations of two canonically conjugate variables has a value less than the lower limit given by $\hbar/2$.

The real problem regarding the interpretation of the uncertainty principle crops up if one recalls Heisenberg's famous thought experiments illustrating the uncertainty principle (Heisenberg 1927, 1930). Heisenberg's analysis of those thought experiments, adopted in many standard treatments on quantum mechanics (for example Bohm 1951, Pauli 1958, Blokhinstev 1964), implies the following meaning of the uncertainty principle:

(ii) The product of the uncertainties in the simultaneous measurements of two canonically conjugate variables of a single particle has a lower bound of the order of h.

In the above statement, the uncertainty in a single measurement is interpreted as the estimate of imprecision in the value of a dynamical variable inferred from the measuring device. On the face of it, operational meanings of (i) and (ii) seem to be basically different. This naturally raises the question: Are the two statements (i) and (ii) equivalent in the sense that one implies the other? There are contradictory viewpoints in the literature on this point. For example, Margenau (1950, 1963) argues that (ii) is operationally untenable while Jammer (1974) contends that (i) and (ii) are essentially equivalent subject to a certain measurement-theoretical assumption. However, it is clear that unless (i) and (ii) are shown to be equivalent, Heisenberg's thought experiments cease to be meaningful because then they cannot be regarded as illustrations of the rigorous form of the uncertainty relation implied by the basic formalism of quantum mechanics. A careful investigation is needed to settle this issue, and we shall take it up in a future communication.

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